where

$$N = \frac{m}{1-p}\, p^{\frac{n}{r}}, \text{ and } \lambda N = \lambda \frac{m}{1-p} + n\lambda \frac{p}{r};$$

or if p be greater than 1,

$$\lambda . y_{a+n} = \lambda y_a + \frac{m}{p-1} - N, \text{ where } \lambda N = \lambda \frac{m}{p-1} + n\lambda \frac{p}{r}.$$

By way of illustrating the use of this formula, Mr. Meikle shows how by its means Mr. Jellicoe's Table of Indian Rates (*see* vol. i., page 169, of this *Journal*) is constructed, and concludes this part of his paper by observing that the other modifications proposed by Mr. Gompertz are not generally adopted because the values obtained by them will not, in any instance, correspond with those of the original table, and that the series can be continued only by a repetition of the process and the use of fresh constants, in which case there will still be a slight break in the harmony of it. Mr. Galloway (says Mr. Meikle) admits that he was obliged "to vary the numbers before and after the middle age, in order to maintain the regularity of the progression in passing from the first formula to the second."

The author then proceeds to describe the method of differences, citing Mr. Farren's use of it in his *Life Contingency Tables*, and suggesting certain modifications of his own, for information as to which the reader is referred to Mr. Meikle's manuscript.

A Problem in Fire Insurance—To apportion a given Loss on Property insured by Specific Policies. By THOMAS MILLER, *Esq., of the Scottish Union Insurance Society, London.*

A SPECIFIC POLICY is one which insures stated sums on particular risks, and guarantees the assured against any loss thereon within the limits of the amounts insured. It differs from an AVERAGE or FLOATING POLICY in several respects—the latter, amongst other privileges, having only to bear a share of any loss compared with the whole loss, similar to the proportion which the amount it insures bears to the whole value of the property insured at the time of the fire.

In solving this problem it requires to be noticed that, by the conditions of insurance, when any stated risk is insured by more

than one specific policy, each one "*shall be liable to a payment only of a rateable proportion of any loss or damage which may be sustained:*" that is, the loss shall be apportioned amongst the policies in proportion to the amounts insured by them on the specified property.

If each one of two or more policies insure A SUM on A CERTAIN SPECIFIED PROPERTY, the insurances are said to be CONCURRENT with each other. But if, while one or more of these sums insure such property, others of them insure only part of it, or perhaps cover the whole of it *and some other property in addition,* then the insurances are NOT CONCURRENT with each other. Thus, policies covering stock alone are concurrent; but they are not concurrent with others which insure A SUM on stock and fixtures, neither is the insurance on A BUILDING concurrent with another on A FLOOR of it.

To divide a loss between CONCURRENT INSURANCES is quite simple. As each will bear its share of it in proportion to the amount which it insures, nothing further is required than to work the question by the common rule of distributive proportion.

With NON-CONCURRENT policies, however, it is not so easy to apply this rule of proportion; and yet the conditions of the policies make it imperative that it should be done. As policies which insure respectively stock, utensils and fixtures, stock and fixtures, and stock alone, cannot, from being different in kind, be made proportional to each other, unless some way be found of separating them into elementary parts, and putting similar elementary divisions into proportion with each other, it follows that, without *some adjustment,* the rule cannot be applied at all. It will however be afterwards seen that an adjustment of the insurances may be made, and thus the correct solution of the question be easily obtained.

But a further complication of the question arises from the circumstance that the assured shall bear no part of any loss so long as a specific policy insuring the kind of property lost by fire shall remain unexhausted. The loss in the first instance must be met by the Insurance Companies, and it is only the excess of loss over the sums insured which the persons insured have to bear themselves.

But again, the assured cannot favour one Office, or make one pay more than its proper share of loss. This follows from the fact that the policies define the limits of the risks incurred, state the sums insured, and afford the term "rateable proportion" as the rule of apportionment; and as it is necessary that all these should

be kept in view, we shall here state them at length for the purpose of future reference; viz.—

Conditions of apportionment.

1. Losses shall be borne proportionally to the amounts insured on the kinds of property claimed for.
2. The amounts insured are defined and limited by the policies themselves.
3. The assured shall bear no part of a loss while any policy insuring the kind of property destroyed is unexhausted by the loss.

We now proceed to draw our conclusions from these premises: and first it may be noticed, that if a policy insure A SUM on STOCK ONLY, and another A SUM on STOCK AND UTENSILS, and if a loss occur ON STOCK AND ON UTENSILS—then, as each policy insures STOCK, each must bear a share of the loss thereon, proportioned to the amount which it insures on STOCK (*see* 1st Condition); and as the second policy alone insures utensils, IT must bear the whole loss on UTENSILS; but as the amount which it insures is defined and limited (*see* 2nd Condition), it follows that said amount must consist of two parts, whereof one is applicable to bear a loss on stock in proportion with the sum insured by the other policy, and the other is applicable to bear the loss on utensils.

1*st Deduction.*—In a similar way it may be shown that if any policy insure A SUM on two or more kinds of property, that sum may BE DIVIDED into elementary parts similar to the divisions of other policies insuring the same or part of the same property.

2*nd Deduction* (derived from 1st Condition).—If the sum insured and the loss paid by any policy be given, and if THE LOSSES PAID by other policies on the same property be known, THE SUMS INSURED by them may be found by simple proportion. Thus: let A insure £100 on stock and pay thereon a loss of £50—if B pay a loss on same stock to the extent of £75, and C to the extent of £100, it is because they respectively insure £150 and £200 on same stock.

3*rd Deduction* (also derived from 1st Condition).—A SUM which bears a share of loss proportioned to the loss on same property borne by another sum must insure that same property, and nothing else. It is, for example, because a sum insures FIXTURES and fixtures ALONE, that it can bear a loss PROPORTIONALLY with a sum insuring FIXTURES ALONE.

Having seen by first and last deductions that the amounts

insured by the more general non-concurrent policies MUST be subdivided, in order that the condition of proportional payments of loss may be complied with, we may now consider how this is to be done.

As each policy is complete IN ITSELF, the sums which it insures cannot depend upon the terms of any other, but must be got from itself alone, AS IF NO OTHER INSURANCE EXISTED. Now the sum which each policy insures (if no other insurance existed) would be applicable to pay any loss on the property insured, and each part of that sum would be similarly liable for its share of the loss with every other equal part. Wherefore it follows, that if the loss be subdivided the sum insured by the policy will follow a similar law and be subdivided in like proportions, and, so far as that loss is concerned, the liability of each subdivision of the sum insured will be discharged on paying its proportion of the loss; and as the existence of other insurances does not affect the terms of any policy, the proportions will remain the same even although other insurances do exist. Loss, therefore, FIXES the subdivisions of the sum insured, such subdivisions being variables which cannot receive fixed values until the occurrence of definite loss.

This method harmonizes with the implied intention under which policies are effected insuring in one sum more divisions of risk than others; namely, that the subdivisions shall remain unknown until loss occur to determine them. The assured reaps considerable advantages from his policies being general in their terms.

Having now found the rule by which the amounts insured by policies may be subdivided, it should here be stated that the 3rd Condition may sometimes call for a farther adjustment of the results. It is true that the sums insured by one policy cannot depend upon those insured by another; but, nevertheless, the assured must be no loser on any part of the property insured while a specific policy thereon is unexhausted. Should it therefore appear, as it sometimes does, when non-concurrent policies have to bear the loss amongst them, that the division of sums proportionally to the subdivisions of the loss would make the assured a loser on one or more divisions of risk, while on others the whole sum insured would not be exhausted, then the policies which include such divisions of risk must make up the excess of loss thereon: and as *proportion* is the rule of apportionment, and the total amount insured by each cannot vary, this must be done proportionally to the amount each insures, and the remainders of these

amounts must, as formerly, be divided proportionally to the other subdivisions of loss.

The method of apportionment now deduced may be summed up thus:—

SUMMARY.

I. *To divide a sum insured into subdivisions.*

1. Take the losses on the subdivisions of property, and divide the sum insured into like proportions.
2. When by such subdivisions the assured would be a loser on property covered by unexhausted specific policies, then the policies insuring that particular property must make up the excess of loss thereon proportionally to the amounts they insure on same, and the remainder of the sums insured by them must be divided in proportion to the losses on the other subdivisions of risk.

II. *To apportion the loss.*

Divide the amount of loss on each subdivision of risk amongst the policies so as to be proportioned to the sums insured thereon by those of smallest range, and to the subdivisions of sums applicable thereto of those of greater range.

The preceding method of apportionment will, on examination, be found in harmony with the three conditions of apportionment already referred to. The losses are proportional to the sums applicable to pay them; the sums insured do not vary in amount from those stated in the policies; and the assured bears no loss while any policy on the property insured is unexhausted. Further, the plan is of UNIVERSAL APPLICATION—for if the loss under non-concurrent policies be greater, equal to, or less than the amount insured by any policy or policies, it furnishes the means of dividing the insurances into their elementary parts, or, in other words, of ascertaining the sums applicable to bear proportionate shares of loss: and this is all that is required; for, having found these, the question becomes one of distributive proportion, which can be worked in practice by the common rules of arithmetic. It may also be noticed that this plan, IF APPLIED TO CONCURRENT POLICIES, as it may be, YIELDS THE SAME RESULTS AS BY THE COMMON METHOD, so that there is the most convincing proof of its adaptation to every case. It is therefore evident that, while consistent with the conditions of insurance, it is also based upon the strictest equity.

SALVAGE.—When the values of the salvage have been ascer-

tained, the proportions to be repaid may be found in the following manner:—

Apportion the loss, *less salvage*, in the manner previously pointed out: then the differences between the sums so found and those previously paid by the different insurances on the same property will be the amounts to be repaid as salvage.

Various other plans have been adopted in allocating losses under non-concurrent policies, but before noticing them further it may be well to exemplify the operation of the method now propounded by one or two practical cases.

Let Office A's policy insure—
On dwelling £100
On warehouse £100
——£200*

And Office B's policy insure—
On dwelling and warehouse, without division of sum . £200†

Loss—
On dwelling £150
On warehouse £50
——£200‡

A's policy insures £100 on EACH part of the risk; and we find by the Summary, 1st part (1), that the amount insured by B's policy is divisible into parts proportional to the losses on the two divisions of risks. So that the divisions of that policy applicable to bear the loss will be, £150 on dwelling and £50 on warehouse. Wherefore the following is the manner in which the loss should be apportioned:—

	Insured by		Loss.	Loss paid by	
	A.	B.		A.	B.
	£.	£.	£.	£.	£.
Dwelling .	100	150	150	60	90
Warehouse	100	50	50	$33\frac{1}{3}$	$16\frac{2}{3}$
	£200*	£200†	£200‡	£$93\frac{1}{3}$	£$106\frac{2}{3}$

Here it will be seen that the amounts held to be insured coincide with those fixed by the policies; that the losses are borne proportionally to the sums applicable to bear them; and that the excess of loss which B pays over A is consequent upon the divisions of sums in the former case depending upon the loss for fixed values, and in the latter being fixed by the terms of the policy itself.

Take another example of apportionment:—

Let Office A insure £1,000* on *m* and *n*.
Office B „ £1,000† on *n* and *o*.
Office C „ £1,000‡ on *o* and *p*.
Loss on *m*, £500; on *n*, £500; on *o*, £1,000=£2,000.§

A's policy—*see* Summary, 1st part (1)—would be divided into the following sums applicable to bear proportionate shares of loss; viz.,

£500 on *m* and £500 on *n*.
B's thus: £333⅓ on *n* and £666⅔ on *o*.
C's thus: £1,000 on *o* and nothing on *p*.

Placing these figures in a tabular form, along with the losses, and dividing proportionally, we obtain the following results:—

	Sums applicable to pay Loss.			Loss.	Loss paid by		
	A.	B.	C.		A.	B.	C.
	£.	£.	£.	£.	£.	£.	£.
m ..	500	..	..	500	500		
n ..	500	333⅓	..	500	300	200	
o ..	..	666⅔	1,000	1,000	..	400	600
	£1,000*	£1,000†	£1,000‡	£2,000§	£800	£600	£600

In this case also it will be found that all the conditions of apportionment have been complied with.

One other example may be given, to show how a loss may be apportioned, when, after dividing the amounts insured into elementary parts, it is found that the assured would be a loser, while an unexhausted policy would be liable to pay the loss.

Let the sums insured by the different Offices be the same as in last example, but let the *losses* be—on *m*, £500; on *n*, £500; on *o*, £1,000; and on *p*, £500: together, £2,500. In this case the separate elementary parts of each policy would, on trial, be found to take the following proportions:—

	Amounts applicable to bear Loss.			Totals.
	A.	B.	C.	
	£.	£.	£.	£.
m ..	500	..	..	500
n ..	500	333⅓	..	833⅓
o ..	..	666⅔	666⅔	1,333⅓
p ..	..	..	333⅓	333⅓
	£1,000	£1,000	£1,000	£3,000

If so divided, however, the assured would lose £166⅔ on *p*, while C's policy insuring that property would not be exhausted. Wherefore—*see* Summary, 1st part (2)—the amounts must be adjusted so as to make C's policy bear the whole loss of £500 on *p*; and then the claim will be apportioned thus :—

	Sums applicable to bear Loss.			Loss.	Loss paid by		
	A.	B.	C.		A.	B.	C.
	£.	£.	£.	£.	£.	£.	£.
m ..	500	..	..	500	500		
n ..	500	333⅓	..	500	300	200	
o ..	..	666⅔	500	1,000	..	571*	429*
p ..	..	..	500	500	..		500
	£1,000	£1,000	£1,000	£2,500	£800	£771	£929

* Neglecting fractions.

These examples are sufficient to indicate the method to be pursued in apportioning losses under non-concurrent policies. It would be easy to solve any case which might occur in practice, and it is unnecessary to extend this article by giving further illustrations.

Some advantage, however, may be derived from contrasting other plans of apportionment with this one.

I. It has been maintained by some that policies insuring a sum on A PORTION of the property covered by others should, in the first place, bear the loss, leaving only the excess above the amounts they insure to be borne by the others. Thus, £100 insured on stock would first be exhausted, and only the excess of loss above that sum would be borne by a policy which insured a sum on the stock *and* fixtures. But, as EACH policy insures STOCK, the loss must be proportioned between them according to the amounts they insure on STOCK (*see* 1st Condition); so that this method of apportionment cannot be correct.

II. Another plan which has been advocated is, to make the WHOLE AMOUNT INSURED by a more general policy applicable to pay loss on EACH division of the risk insured by it. But as losses are borne proportionally to the sums respectively insured on the property destroyed, this plan assumes that the amounts which more general policies insure greatly exceed those which these policies themselves admit.

Thus, if £1,000 were insured on stock, utensils, and fixtures by one policy, £500 on stock by another, £100 on utensils by a

third, and £200 on fixtures by a fourth, these amounts would be applicable to pay a loss in the following proportions:—

	1st Policy.	2nd Policy.	3rd Policy.	4th Policy.
	£.	£.	£.	£.
Stock	1,000	500		
Utensils	1,000	..	100	
Fixtures..	1,000	..	..	200
	£3,000	£500	£100	£200

The first policy would pay AS IF it insured £1,000 on stock, £1,000 on utensils, and £1,000 on fixtures, together £3,000; while the utmost amount really insured by it is only £,1000.

III. A third plan, now generally adopted, is a modification of the last one. It first makes a policy insuring a sum on two or more divisions of property pay a loss on one of them, along with sums insuring that division alone; then, deducting the sum so paid by it from the sum insured, it makes the remainder bear a loss on another division, along with the sums which insure that division: and this process is continued while a policy insuring any of the divisions has to pay a loss.

This plan, like the previous one, exaggerates in no small degree the amounts insured by the more general policies, and, what is worse, leaves undecided the question it should settle—namely, what sums each policy should pay; for there may be *many different solutions of a question.* In as many ways as the *order* of the divisions can be changed, according to the theory of permutations, there may be as many *conflicting* results of the question; and there is no reason why one should be preferred to another. *In principle* they are all alike.

The following case will exemplify this method:—

Office A's policy insures £100 on dwelling
„ B's „ £100 on store
„ C's „ £200 on dwelling and store

£400

Loss—on dwelling, £225; on store, £75=£300.

Here the divisions of sums are on the *dwelling* and on the *store.* Consequently there are *two* permutations, and *two* conflicting results can be obtained by changing the *order* of the divisions.

First.—C will pay AS IF it insured £200 on dwelling, and £50

on store—together, £250. And the loss will be divided thus: A, £75; B, £50; C, £175 = £300.

Second.—C will pay AS IF its policy insured £200 on store, and £150 on dwelling—together, £350. And the loss will be divided thus: A, £90; B, £25; C, £185 = £300.

As each of these solutions is as good as the other, it is plain that the question remains unsettled. The results, as deduced from the Summary given in this paper, would have been—C's policy applicable in the following proportions to pay the loss: on dwelling, £150, and on store, £50; together, £200, as in the policy. And the loss would have been apportioned thus: A to pay £90; B, £50; C, £160 = £300.

These plans do not exhaust all the methods of apportionment which have been tried, nor is it necessary that all should be referred to. There are different ways of testing whether any one is correct, but *in general* this may be ascertained by reference to the results. If these be tried by the 2nd Deduction, and found to lead to different amounts being insured than those which the policies admit, the erroneous nature of the premises on which the calculation rests is quite certain.

Observations on the Paper by Mr. Jellicoe, published in No. XXII. of this Journal. By* HENRY WILBRAHAM, *Esq., Fellow of Trinity College, Cambridge.*

TAKE first the case in which A has not a life interest, but one for n years, remainder to B. Let P be a sum of consols, $100-\delta$ the present price of consols; consequently, P consols sells for $P\frac{100-\delta}{100}$ cash, which may be called M; A's and B's shares of it being M_1 and M_2.

B has a right to a sum of P consols n years hence: it matters not to him how this sum arises, provided it be ready for him in n years' time. Suppose such a sum now appropriated to accumulate for n years as will produce P, B will be in the same position as if A enjoyed the sum intermediately; and if such sum be now

* "On the Valuation of Property held for Life and in Reversion; and on the due Apportionment of it, when so held on the same Life, between the Tenant for Life and the Remainder-man."